

Comment on “Gravity Waves, Chaos, and Spinning Compact Binaries”

Levin has shown that spinning compact binaries can be chaotic at second post-Newtonian order. However, when higher order dissipational effects are included, the dynamics will no longer be chaotic, though the evolution may still be unpredictable in a practical sense. I discuss some of the additional work that needs to be done to decide how this unpredictability might affect gravitational wave detectors such as LIGO.

Levin [1] has established that the two body problem, with spin, is chaotic at second Post-Newtonian (2PN) order. This important result supports and extends Suzuki and Maeda’s [2] demonstration that the system is chaotic in the test particle limit. The 2PN equations of motion are conservative, and the system is chaotic in a strict mathematical sense. Indeed, the full two body problem conserves total energy [3], but it is natural to treat the flux of gravitational waves as an energy loss mechanism, and the orbital dynamics as dissipative. The first dissipative terms in the orbital dynamics show up at 2.5PN order. To be precise, the orbital dynamics is damped and weakly driven, since some of the gravitational waves backscatter off the spacetime curvature and are re-absorbed by the orbiting bodies. However, the energy loss always exceeds the energy re-absorbed. Consequently, the phase space has a simple attracting fixed point, and is not formally chaotic.

The final section of Levin’s paper deals with the effect of dissipation on the system. Levin notes that “some orbits will sweep through the chaotic region of phase space as they inspiral” and that “dissipation does not obliterate the chaos”. An apparently fractal basis boundary is displayed in Fig. 5 to support this point. However, since there are no unstable quasi-periodic orbits in a damped system, the boundaries can not be truly fractal. If we were to “zoom in” on a small portion of the boundary we would see that it is actually smooth. Thus, in a strict mathematical sense, the boundaries are not fractal and the dynamics is not chaotic. I believe that Levin is using the term “fractal scaling” in the way it is used to describe real objects such as trees or coastlines. These scaling laws only hold over a finite range of length scales. For the two body problem it would be interesting to study how the number of decades of fractal scaling varies with the strength of the dissipation. The larger the range of effectively fractal scaling, the more unpredictable the system is in a practical sense.

The situation here is reminiscent of the problems encountered when studying the quantum analogs of classically chaotic systems and the recovery of chaos in the classical limit. It was argued that quantum systems could not be chaotic since the uncertainty principle does not allow there to be fractal structures in phase space. Nevertheless, a shadowy imprint of the classical fractal structure was found to remain [4], and it is now recognized that energy levels in the quantum analogs of classically

integrable systems differ from those in classically chaotic systems [5]. Perhaps something similar applies here. As Frankel and I argued earlier [6], the dissipative dynamics can be approximated by a sequence of conservative orbits with decreasing energy and angular momentum. The orbits of the dissipative system are effectively sewn together from the threads of a conservative chaotic system. Consequently, the behaviour of the dissipative system can be erratic and difficult to predict, even though the dynamics is not chaotic.

The most important questions still to be addressed are how prevalent and how strong the erratic behaviour might be, and what effect this might have on the detection of gravitational waves. The question of prevalence can be addressed by looking at how much of the phase space of the non-dissipative dynamics is stochastic, and seeing how much of the inspiral is spent in the stochastic regions. The effect that passage through a stochastic region has on gravitational wave templates can be estimated by comparing the Lyapunov timescale τ_λ of these regions to their dissipative timescale τ_d [7]. If $\tau_\lambda \ll \tau_d$, then the unpredictability will be pronounced and gravitational waveforms for nearby trajectories will evolve very differently, as pointed out in Refs. [6,1]. If passage through a stochastic region happened to coincide with passage through the LIGO frequency range, the gravitational wave signal would be very difficult to extract.

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PACS numbers: 04.30.Db, 95.30.Sf, 97.60.Jd, 97.60.Lf

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[7] Note, while τ_λ and τ_d are both coordinate dependent, their ratio is a coordinate independent quantity.